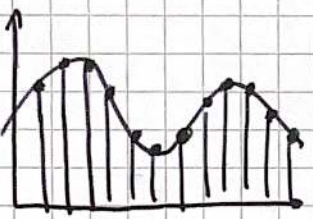
MMET-Einführung

• Matlab Einführung

Signale u. Systeme

Signal: physikalische Darstellung von Information

(z.B. Sprachsignal $s(t)$, $s: \mathbb{R} \to \mathbb{R}$)
(z.B. Bildsignal $s(x,y)$, $s: \mathbb{R}^2 \to \mathbb{R}$)
(z.B. Videosignal $s(x,y,t)$, $s: \mathbb{R}^3 \to \mathbb{R}$)



Abtastung "Sampling" → $f[n]$ diskretes Signal
→ abzählbare Folge aus kontinuierlichem Signal $s(t)$
äquidistante Zeitpunkte nT_s, $n \in \mathbb{N}/\mathbb{Z}$
T_s Abtastintervall
$r = \frac{1}{T_s}$ Abtastrate

(22,7 μs, 44,1 kHz → CD)

A/D-Umsetzung (analog zu digital)

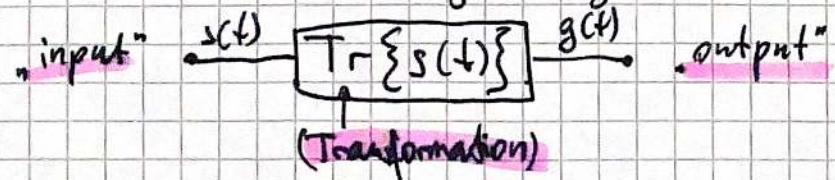
1. Abtastung $s(t) \to s(nT_s) = f[n]$ → zeitdiskret
2. Quantisierung: Rundung → wertediskret
3. binäre Codierung

Signalvektor

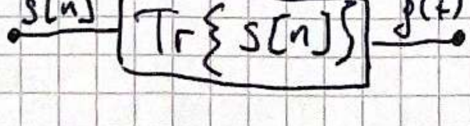
$$f = \begin{bmatrix} f[1] \\ f[2] \\ \dots \\ f[N] \end{bmatrix}$$ (N Datenpunkte)

System: Übertragung, Umwandlung, Speicherung, Verarbeitung von Signalen

Verstärker: $g(t) = a \cdot s(t)$, $a \in \mathbb{R}^{>1}$



kontinuierlich / diskret

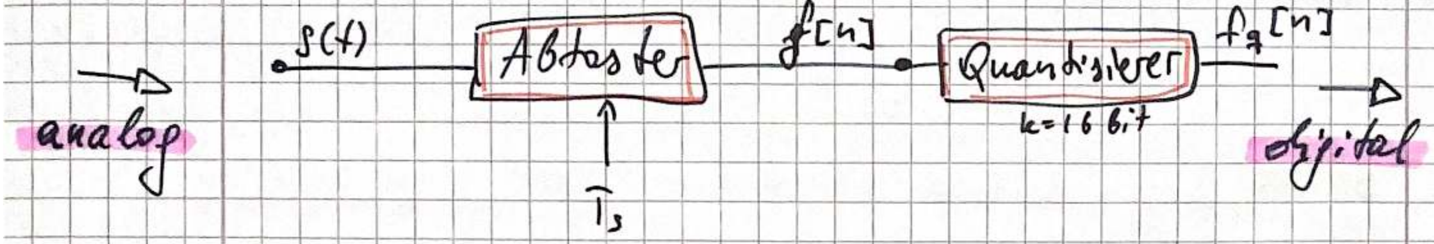


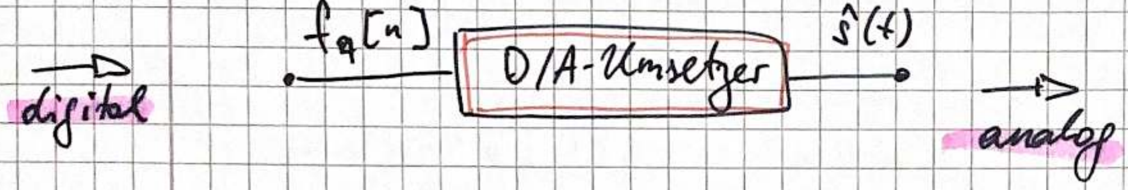
DSP: digital signal processor

$g[n] = a \cdot s[n]$, $a \in \mathbb{R}^{>1}$

Signale u. Systeme – forts.

Blockdiagramm:



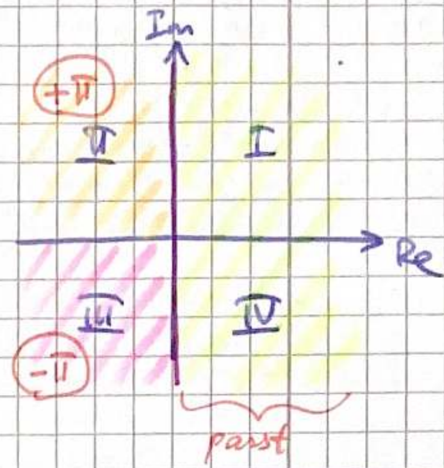


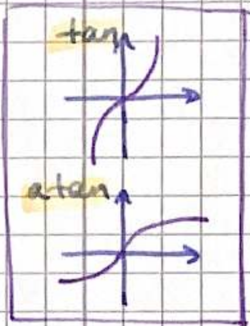
Komplexe Zahlen

kartesisch → polar

$$r = |z| = \sqrt{x^2 + y^2}$$

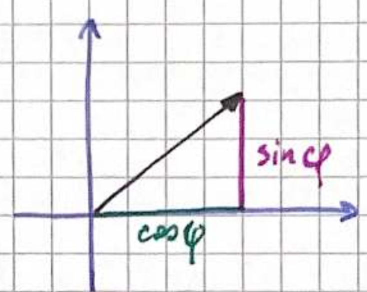
$$\varphi = \begin{cases} \arctan\left(\frac{y}{x}\right) & x > 0 \\ \arctan\left(\frac{y}{x}\right) + \pi & x < 0,\ y \geq 0 \\ \arctan\left(\frac{y}{x}\right) - \pi & x < 0,\ y < 0 \\ -\frac{\pi}{2} & x = 0,\ y > 0 \\ +\frac{\pi}{2} & x = 0,\ y < 0 \end{cases}$$





polar → kartesisch

$$x = r \cdot \cos(\varphi)$$
$$y = r \cdot \sin(\varphi)$$



$$e^{j\varphi} = \underbrace{\cos(\varphi) + j\sin(\varphi)}_{cis(\varphi)}$$

Formeln

$e^{j0} = 1$ $\quad e^{j\pi} = -1$ $\quad e^{j\pi/2} = j$ $\quad e^{-j\pi/2} = -j$

$\frac{1}{j} = -j$ $\quad \frac{1}{z} = z^{-1} = \frac{1}{r} \cdot e^{-j\varphi}$ $\quad z \cdot z^* = |z|^2 = x^2 + y^2 = r^2$

$$e^{+j\alpha} + e^{-j\alpha} = 2\cos(\alpha) = 2 \cdot Re\{e^{j\alpha}\}$$
$$e^{+j\alpha} - e^{-j\alpha} = 2j \cdot \sin(\alpha) = 2j \cdot Im\{e^{j\alpha}\}$$

$$\cos(\alpha) = \frac{1}{2}\left(e^{+j\alpha} + e^{-j\alpha}\right)$$
$$\sin(\alpha) = \frac{1}{2j}\left(e^{+j\alpha} - e^{-j\alpha}\right)$$

Sinussignale

komplexe Signale

$z(t) = A \cdot e^{j(\omega t + \varphi)}$

Amplitude (A), Phase (φ)

$z(t) = \underbrace{A \cdot e^{j\varphi}}_{X} \cdot e^{j\omega t}$

$e^{j\omega t}$: rotierender Anteil

X: Phasor = komplexe Amplitude

reele Signale

$s(t) = A \cdot \cos(\omega t + \varphi)$

$s(t) = Re\{A \cdot e^{j(\omega t + \varphi)}\}$

$s(t) = Re\{X \cdot e^{j\omega t}\}$

$s(t) = \frac{1}{2} \cdot X \cdot e^{j\omega t} + \frac{1}{2} \cdot X^* \cdot e^{-j\omega t}$

Formeln

$\sin(\alpha) = \cos(\alpha - \pi/2)$ | $\cos(\alpha) = \sin(\alpha + \pi/2)$

$\sin(\alpha) = -\sin(-\alpha)$ | $\cos(\alpha) = \cos(-\alpha)$

ungerade | gerade

$\sin(\pi k) = 0 \quad \forall k \in \mathbb{Z}$ | $\cos(2\pi k) = 1 \quad \forall k \in \mathbb{Z}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\omega = 2\pi \cdot f = \frac{2\pi}{T}$

ω: Kreisfrequenz, f: Frequenz, T: Periodendauer

Rechnen mit Sinussignalen

Addition

- → gleiche Kreisfrequenz ω erforderlich!
- → alles in cos umwandeln

$$s(t) = s_1(t) + s_2(t)$$
$$= A_1 \cdot \cos(\underline{\omega} t + \varphi_1) + A_2 \cdot \cos(\underline{\omega} t + \varphi_2)$$
$$= \mathrm{Re}\{A_1 \cdot e^{j\underline{\omega} t} \cdot e^{j\varphi_1} + A_2 \cdot e^{j\underline{\omega} t} \cdot e^{j\varphi_2}\}$$
$$= \mathrm{Re}\{e^{j\underline{\omega} t} (A_1 \cdot e^{j\varphi_1} + A_2 \cdot e^{j\varphi_2})\}$$
$$= \mathrm{Re}\{e^{j\underline{\omega} t} (\underline{X}_1 + \underline{X}_2)\}$$
$$= \mathrm{Re}\{e^{j\underline{\omega} t} \cdot \underline{X}\}$$
$$= \mathrm{Re}\{e^{j\underline{\omega} t} \cdot A \cdot e^{j\varphi}\}$$
$$= A \cdot \cos(\underline{\omega} t + \varphi)$$

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